V11.0

ECONOMETRÍA APLICADA

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S1

ANÁLISIS UNIVARIANTE DE SERIES TEMPORALES

REPRESENTACIONES PSI (MA) - PI (AR)

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1. DIVISIÓN DE POLINOMIOS: EJEMPLOS Y REGLAS GENERALES

■ $$\frac{1-0.8B}{1+0.6B} = \psi(B),\ \text{con} \qquad [1]$$
$$\psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \psi_3 B^3 + \cdots = \sum_{i=0}^{\infty} \psi_i B^i \ \ (\psi_0 = 1).$$

$$[1] \Rightarrow \quad \begin{gathered}(1+0.6B)(1+\psi_1 B + \psi_2 B^2 + \cdots) = 1 - 0.8B,\\ 1 + \psi_1 B + \psi_2 B^2 + \cdots + 0.6B + 0.6\psi_1 B^2 + 0.6\psi_2 B^3 + \cdots = 1 - 0.8B.\end{gathered} \qquad [1.1]$$

$$[1.1] \Rightarrow \quad \begin{gathered}\begin{aligned}
B &: \psi_1 + 0.6 = -0.8 &&\Rightarrow \psi_1 = -1.4,\\
B^2 &: \psi_2 + 0.6\psi_1 = 0 &&\Rightarrow \psi_2 = -0.6\psi_1 = 0.84,\\
B^3 &: \psi_3 + 0.6\psi_2 = 0 &&\Rightarrow \psi_3 = -0.6\psi_2 = -0.504,
\end{aligned}\\ \vdots \\
\psi_1 = -1.4, \psi_i = -0.6\psi_{i-1} \ \text{para } i \geq 2,\ \text{o bien}\\
\psi_i = -1.4 \times (-0.6)^{i-1} \ \text{para } i \geq 1.\end{gathered} \qquad [1.2]$$

$$\frac{1-0.8B}{1+0.6B} = 1 - 1.4B + 0.84B^2 - 0.504B^3 + 0.3024B^4 - 0.18144B^5 + \cdots$$ ■

■ ■ $$\frac{1+0.6B}{1-0.8B} = \pi(B),\ \text{con} \qquad [2]$$
$$\pi(B) = 1 - \pi_1 B - \pi_2 B^2 - \pi_3 B^3 - \cdots = -\sum_{i=0}^{\infty} \pi_i B^i \ \ (\pi_0 = -1).$$

$$[2] \Rightarrow \quad \begin{gathered}(1-0.8B)(1-\pi_1 B - \pi_2 B^2 - \cdots) = 1 + 0.6B,\\ 1 - \pi_1 B - \pi_2 B^2 - \cdots - 0.8B + 0.8\pi_1 B^2 + 0.8\pi_2 B^3 + \cdots = 1 + 0.6B.\end{gathered} \qquad [2.1]$$

$$[2.1] \Rightarrow \quad \begin{gathered}\begin{aligned}
B &: -\pi_1 - 0.8 = 0.6 &&\Rightarrow \pi_1 = -1.4,\\
B^2 &: -\pi_2 + 0.8\pi_1 = 0 &&\Rightarrow \pi_2 = 0.8\pi_1 = -1.12,\\
B^3 &: -\pi_3 + 0.8\pi_2 = 0 &&\Rightarrow \pi_3 = 0.8\pi_2 = -0.896,
\end{aligned}\\ \vdots \\
\pi_1 = -1.4, \pi_i = 0.8\pi_{i-1} \ \text{para } i \geq 2,\ \text{o bien}\\
\pi_i = -1.4 \times 0.8^{i-1} \ \text{para } i \geq 1.\end{gathered} \qquad [2.2]$$

$$\frac{1+0.6B}{1-0.8B} = 1 + 1.4B + 1.12B^2 + 0.896B^3 + 0.7168B^4 + 0.57344B^5 + \ldots$$ ■ ■

2. MODELOS ARMA: REPRESENTACIONES PSI (MA) (WOLD) - PI (AR)

$$\phi(B)Y_t = \mu + \theta(B)A_t\,,$$
$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p\,,$$
$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q\,,$$

con todas las raíces de $\phi(x) = 0$ y $\theta(x) = 0$ fuera del círculo unitario.

$$Y_t = \frac{\mu}{\phi(B)} + \frac{\theta(B)}{\phi(B)} A_t = \beta_0 + \frac{\theta(B)}{\phi(B)} A_t\,, \text{ con } \beta_0 = \frac{\mu}{\phi(1)}\cdot$$

$$\frac{\theta(B)}{\phi(B)} = \psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \ldots = \sum_{i=0}^{\infty} \psi_i B^i \;\; (\psi_0 = 1), \qquad [3]$$

$$Y_t = \beta_0 + \psi(B)A_t = \beta_0 + A_t + \psi_1 A_{t-1} + \psi_2 A_{t-2} + \ldots \Rightarrow \psi_i = \frac{\partial Y_t}{\partial A_{t-i}} \;\; (i = 0, 1, 2, \ldots).$$

$$\frac{\phi(B)}{\theta(B)} Y_t = \frac{\mu}{\theta(B)} + A_t = \mu_0 + A_t\,, \text{ con } \mu_0 = \frac{\mu}{\theta(1)} = \frac{\phi(1)}{\theta(1)} \times \beta_0\,.$$

$$\frac{\phi(B)}{\theta(B)} = \pi(B) = 1 - \pi_1 B - \pi_2 B^2 - \ldots = -\sum_{i=0}^{\infty} \pi_i B^i \;\; (\pi_0 = -1), \qquad [4]$$

$$\pi(B)Y_t = \mu_0 + A_t \Rightarrow Y_t - \pi_1 Y_{t-1} - \pi_2 Y_{t-2} - \ldots = \mu_0 + A_t\,,$$

$$Y_t = \mu_0 + \pi_1 Y_{t-1} + \pi_2 Y_{t-2} + \ldots + A_t \Rightarrow \pi_i = \frac{\partial Y_t}{\partial Y_{t-i}} \;\; (i = 1, 2, \ldots).$$

$$[3] \Rightarrow \qquad \phi(B)\psi(B) = \theta(B). \qquad [5]$$

$$[4] \Rightarrow \qquad \theta(B)\pi(B) = \phi(B). \qquad [6]$$

AR(1): $$(1 - \phi_1 B)Y_t = A_t \Rightarrow \phi(B) = 1 - \phi_1 B,\; \theta(B) = 1.$$

$$(1 - \phi_1 B)(1 + \psi_1 B + \psi_2 B^2 + \ldots) = 1,$$
$$1 + \psi_1 B + \psi_2 B^2 + \ldots - \phi_1 B - \phi_1\psi_1 B^2 - \phi_1\psi_2 B^3 - \ldots = 1.$$

$$[5] \Rightarrow \qquad \begin{aligned} B &: \psi_1 - \phi_1 = 0 \Rightarrow \psi_1 = \phi_1\,, \\ B^2 &: \psi_2 - \phi_1\psi_1 = 0 \Rightarrow \psi_2 = \phi_1\psi_1 = \phi_1^2\,, \\ B^3 &: \psi_3 - \phi_1\psi_2 = 0 \Rightarrow \psi_3 = \phi_1\psi_2 = \phi_1^3\,, \\ &\vdots \end{aligned}$$

$$\psi_i = \phi_1\psi_{i-1} \text{ para } i \geq 1 \text{ (con } \psi_0 = 1) \Rightarrow \psi_i = \phi_1^i \text{ para } i \geq 0.$$

$[6] \Rightarrow$

$$1 - \pi_1 B - \pi_2 B^2 - ... = 1 - \phi_1 B,$$
$$\pi_1 = \phi_1, \pi_i = 0 \text{ para } i > 1.$$

AR(2): $(1 - \phi_1 B - \phi_2 B^2)Y_t = A_t \Rightarrow \phi(B) = 1 - \phi_1 B - \phi_2 B^2,\ \theta(B) = 1.$

$$(1 - \phi_1 B - \phi_2 B^2)(1 + \psi_1 B + \psi_2 B^2 + ...) = 1,$$
$$1 + \psi_1 B + \psi_2 B^2 + ... - \phi_1 B - \phi_1\psi_1 B^2 - \phi_1\psi_2 B^3 -$$
$$-... - \phi_2 B^2 - \phi_2\psi_1 B^3 - \phi_2\psi_2 B^4 - ... = 1.$$

$[5] \Rightarrow$

$$B : \psi_1 - \phi_1 = 0 \Rightarrow \psi_1 = \phi_1,$$
$$B^2 : \psi_2 - \phi_1\psi_1 - \phi_2 = 0 \Rightarrow \psi_2 = \phi_1\psi_1 + \phi_2 =$$
$$= \phi_1\psi_1 + \phi_2\psi_0 \ \ (\psi_0 = 1),$$
$$B^3 : \psi_3 - \phi_1\psi_2 - \phi_2\psi_1 = 0 \Rightarrow \psi_3 = \phi_1\psi_2 + \phi_2\psi_1,$$
$$\vdots$$
$$\psi_i = \phi_1\psi_{i-1} + \phi_2\psi_{i-2} \text{ para } i \geq 2 \text{ (con } \psi_0 = 1, \psi_1 = \phi_1).$$

$[6] \Rightarrow$

$$1 - \pi_1 B - \pi_2 B^2 - ... = 1 - \phi_1 B - \phi_2 B^2,$$
$$\pi_1 = \phi_1, \pi_2 = \phi_2, \pi_i = 0 \text{ para } i > 2.$$

MA(1): $Y_t = (1 - \theta_1 B)A_t \Rightarrow \phi(B) = 1,\ \theta(B) = 1 - \theta_1 B.$

$[5] \Rightarrow$

$$1 + \psi_1 B + \psi_2 B^2 + ... = 1 - \theta_1 B,$$
$$\psi_1 = -\theta_1, \psi_i = 0 \text{ para } i > 1.$$

$$(1 - \theta_1 B)(1 - \pi_1 B - \pi_2 B^2 - ...) = 1,$$
$$1 - \pi_1 B - \pi_2 B^2 - ... - \theta_1 B + \theta_1\pi_1 B^2 + \theta_1\pi_2 B^3 + ... = 1.$$

$[6] \Rightarrow$

$$B : -\pi_1 - \theta_1 = 0 \Rightarrow \pi_1 = -\theta_1,$$
$$B^2 : -\pi_2 + \theta_1\pi_1 = 0 \Rightarrow \pi_2 = \theta_1\pi_1 = -\theta_1^2,$$
$$B^3 : -\pi_3 + \theta_1\pi_2 = 0 \Rightarrow \pi_3 = \theta_1\pi_2 = -\theta_1^3,$$
$$\vdots$$
$$\pi_i = \theta_1\pi_{i-1} \text{ para } i \geq 1 \text{ (con } \pi_0 = -1) \Rightarrow \pi_i = -\theta_1^i \text{ para } i \geq 0.$$

MA(2): $Y_t = (1 - \theta_1 B - \theta_2 B^2) A_t \Rightarrow \phi(B) = 1,\ \theta(B) = 1 - \theta_1 B - \theta_2 B^2.$

[5] $\Rightarrow$
$$1 + \psi_1 B + \psi_2 B^2 + ... = 1 - \theta_1 B - \theta_2 B^2,$$
$$\psi_1 = -\theta_1,\ \psi_2 = -\theta_2,\ \psi_i = 0 \text{ para } i > 2.$$

$$(1 - \theta_1 B - \theta_2 B^2)(1 - \pi_1 B - \pi_2 B^2 - ...) = 1,$$
$$1 - \pi_1 B - \pi_2 B^2 - ... - \theta_1 B + \theta_1 \pi_1 B^2 + \theta_1 \pi_2 B^3 + ... -$$
$$- \theta_2 B^2 + \theta_2 \pi_1 B^3 + \theta_2 \pi_2 B^4 + ... = 1.$$

[6] $\Rightarrow$
$$B : -\pi_1 - \theta_1 = 0 \Rightarrow \pi_1 = -\theta_1,$$
$$B^2 : -\pi_2 + \theta_1 \pi_1 - \theta_2 = 0 \Rightarrow \pi_2 = \theta_1 \pi_1 - \theta_2 =$$
$$= \theta_1 \pi_1 + \theta_2 \pi_0 \quad (\pi_0 = -1),$$
$$B^3 : -\pi_3 + \theta_1 \pi_2 + \theta_2 \pi_1 = 0 \Rightarrow \pi_3 = \theta_1 \pi_2 + \theta_2 \pi_1,$$
$$\vdots$$
$$\pi_i = \theta_1 \pi_{i-1} + \theta_2 \pi_{i-2} \text{ para } i \geq 2 \text{ (con } \pi_0 = -1, \pi_1 = -\theta_1).$$

ARMA(1,1): $(1 - \phi_1 B) Y_t = (1 - \theta_1 B) A_t \Rightarrow \phi(B) = 1 - \phi_1 B,\ \theta(B) = 1 - \theta_1 B.$

$$(1 - \phi_1 B)(1 + \psi_1 B + \psi_2 B^2 + ...) = 1 - \theta_1 B,$$
$$1 + \psi_1 B + \psi_2 B^2 + ... - \phi_1 B - \phi_1 \psi_1 B^2 - \phi_1 \psi_2 B^3 - ... = 1 - \theta_1 B.$$

[5] $\Rightarrow$
$$B : \psi_1 - \phi_1 = -\theta_1 \Rightarrow \psi_1 = \phi_1 - \theta_1,$$
$$B^2 : \psi_2 - \phi_1 \psi_1 = 0 \Rightarrow \psi_2 = \phi_1 \psi_1 = \phi_1 (\phi_1 - \theta_1),$$
$$B^3 : \psi_3 - \phi_1 \psi_2 = 0 \Rightarrow \psi_3 = \phi_1 \psi_2 = \phi_1^2 (\phi_1 - \theta_1),$$
$$\vdots$$
$$\psi_i = \phi_1 \psi_{i-1} \text{ para } i \geq 2 \text{ (con } \psi_1 = \phi_1 - \theta_1) \Rightarrow \psi_i = \phi_1^{i-1} (\phi_1 - \theta_1) \text{ para } i \geq 1.$$

$$(1 - \theta_1 B)(1 - \pi_1 B - \pi_2 B^2 - ...) = 1 - \phi_1 B,$$
$$1 - \pi_1 B - \pi_2 B^2 - ... - \theta_1 B + \theta_1 \pi_1 B^2 + \theta_1 \pi_2 B^3 + ... = 1 - \phi_1 B.$$

[6] $\Rightarrow$
$$B : -\pi_1 - \theta_1 = -\phi_1 \Rightarrow \pi_1 = \phi_1 - \theta_1,$$
$$B^2 : -\pi_2 + \theta_1 \pi_1 = 0 \Rightarrow \pi_2 = \theta_1 \pi_1 = \theta_1 (\phi_1 - \theta_1),$$
$$B^3 : -\pi_3 + \theta_1 \pi_2 = 0 \Rightarrow \pi_3 = \theta_1 \pi_2 = \theta_1^2 (\phi_1 - \theta_1),$$
$$\vdots$$
$$\pi_i = \theta_1 \pi_{i-1} \text{ para } i \geq 2 \text{ (con } \pi_1 = \phi_1 - \theta_1) \Rightarrow \pi_i = \theta_1^{i-1} (\phi_1 - \theta_1) \text{ para } i \geq 1.$$

En modelos AR(p) [$\Rightarrow q = 0,\ \theta(B) = 1$], $\psi(B) = 1/\phi(B)$ es un polinomio infinito, con cada ψ_i $(i \geq 1)$ expresable en función de $\phi_1, ..., \phi_p$, mientras que $\pi(B) = \phi(B)$ es un polinomio finito de grado p con $\pi_i = \phi_i$ $(i = 1, ..., p)$ (Tabla 1).

En modelos MA(q) [$\Rightarrow p = 0,\ \phi(B) = 1$], $\psi(B) = \theta(B)$ es un polinomio finito de grado q con $\psi_i = -\theta_i$ $(i = 1, ..., q)$, mientras que $\pi(B) = 1/\theta(B)$ es un polinomio infinito con cada π_i $(i \geq 1)$ expresable en función de $\theta_1, ..., \theta_q$ (Tabla 1).

En modelos ARMA(p,q), tanto $\psi(B)$ como $\pi(B)$ son polinomios infinitos (Tabla 1), con cada ψ_i y cada π_i $(i \geq 1)$ expresables en función de $\phi_1, ..., \phi_p, \theta_1, ..., \theta_q$.

Las secuencias $\psi_1, \psi_2, ...$ y $\pi_1, \pi_2, ...$ calculadas a partir de cualquier modelo ARMA estacionario e invertible siguen pautas semejantes a las de la ACF teórica y la PACF teórica, respectivamente, del modelo (Tabla 1).

En modelos estacionarios, $\psi_i \to 0$; en modelos invertibles, $\pi_i \to 0$ (Tabla 1).

En general, los coeficientes o pesos ψ_i y π_i en [3]-[4] pueden calcularse recursivamente teniendo en cuenta [5]-[6], de manera que, por un lado,

$$\psi_i = \phi_1\psi_{i-1} + \phi_2\psi_{i-2} + ... + \phi_p\psi_{i-p} - \theta_i \qquad [7]$$

para todo $i > 0$ (donde $\psi_0 = 1$, $\psi_i = 0$ si $i < 0$ y $\theta_i = 0$ si $i > q$), y, por otro lado,

$$\pi_i = \theta_1\pi_{i-1} + \theta_2\pi_{i-2} + ... + \theta_q\pi_{i-q} + \phi_i \qquad [8]$$

para todo $i > 0$ (donde $\pi_0 = -1$, $\pi_i = 0$ si $i < 0$ y $\phi_i = 0$ si $i > p$). Además, de la relación $\psi(B)\pi(B) = 1$ implícita en [3]-[4] se deduce que, para todo $i \geq 1$,

$$\pi_i = -\textstyle\sum_{j=0}^{i-1} \pi_j\psi_{i-j} \ \ (\pi_0 = -1), \ \ \psi_i = \textstyle\sum_{j=0}^{i-1} \psi_j\pi_{i-j} \ \ (\psi_0 = 1) \qquad [9]$$

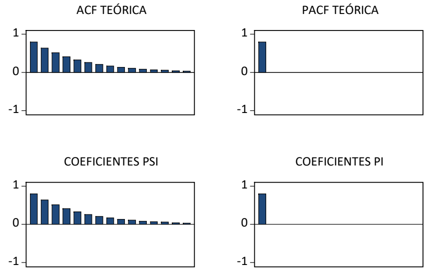
Los coeficientes ψ_i y π_i $(i = 1, 2, ...)$ se pueden calcular en modelos estacionarios e invertibles (en cuyo caso $\psi_i \to 0$ y $\pi_i \to 0$), y también en **modelos no estacionarios** (en cuyo caso la secuencia $\psi_1, \psi_2, ...$ **no** converge a cero) y en modelos **no invertibles** (en cuyo caso la secuencia $\pi_1, \pi_2, ...$ **no** converge a cero). Por ejemplo, en un modelo ARMA(1,1) con $\phi_1 = 1$ (no estacionario), $\psi_i = 1 - \theta_1$ para todo $i \geq 1$, mientras que en un modelo ARMA(1,1) con $\theta_1 = 1$ (no invertible), $\pi_i = \phi_1 - 1$ para todo $i \geq 1$.

Aclaración 1: Cuando $\phi(x) = 0$ tiene al menos una raíz igual a 1, de manera que $\phi(1) = 0$, la constante β_0 en [3] no está definida, lo que requiere considerar la representación PSI de modelos no estacionarios en términos más generales (Sección 3). Cuando $\theta(x) = 0$ tiene al menos una raíz igual a 1, de manera que $\theta(1) = 0$, la constante μ_0 en [4] no está definida y, además, la secuencia $\pi_i = \partial Y_t/\partial Y_{t-i}$ $(i = 1, 2, ...)$ no converge a cero, lo que carece de sentido práctico en muchas ocasiones.

En modelos tanto estacionarios como no estacionarios, los coeficientes $\psi_1, \psi_2, ...$ son útiles para expresar los **errores de previsión** y sus **varianzas** (➪ S4). Por su parte, los coeficientes $\pi_1, \pi_2, ...$ resumen en cualquier modelo la dependencia de una serie con respecto a su propio pasado, lo que suele requerir, desde un punto de vista práctico, la invertibildad del modelo considerado.

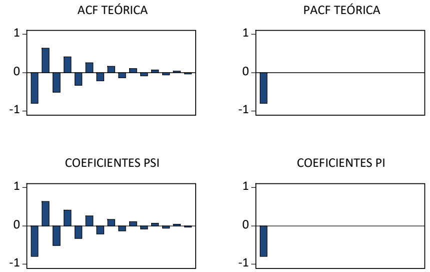
TABLA 1 - MODELOS ARMA ESTACIONARIOS E INVERTIBLES

AR(1)



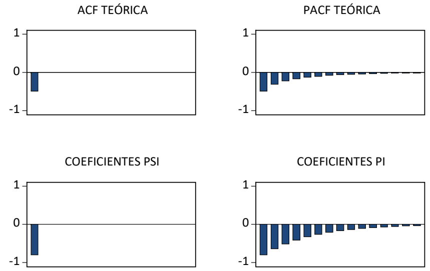
$\phi_1 = 0.8$

AR(1)



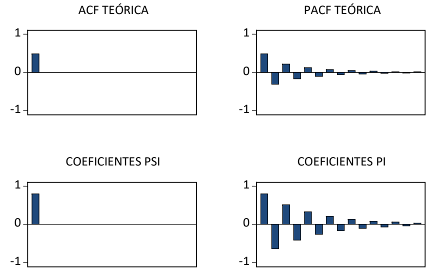
$\phi_1 = -0.8$

MA(1)



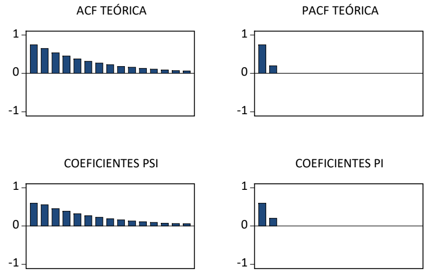
$\theta_1 = 0.8$

MA(1)



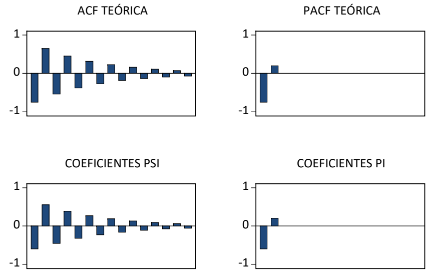
$\theta_1 = -0.8$

AR(2)



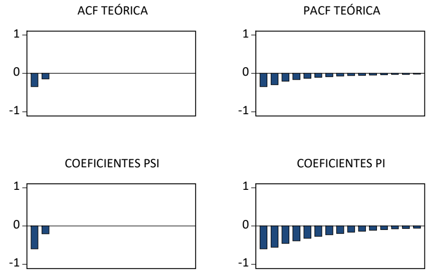
$\phi_1 = 0.6,\ \phi_2 = 0.2$

AR(2)



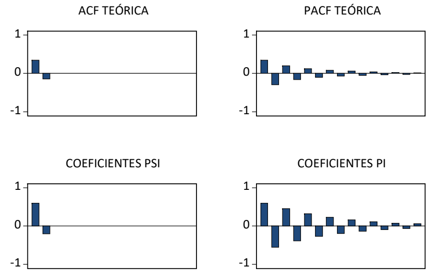
$\phi_1 = -0.6,\ \phi_2 = 0.2$

MA(2)



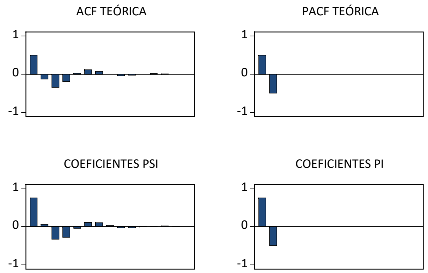
$\theta_1 = 0.6,\ \theta_2 = 0.2$

MA(2)



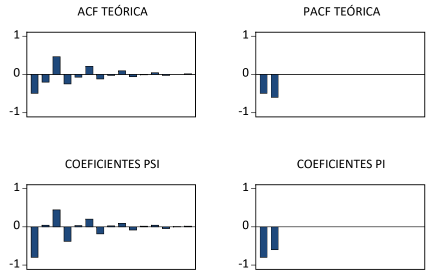
$\theta_1 = -0.6,\ \theta_2 = 0.2$

AR(2)



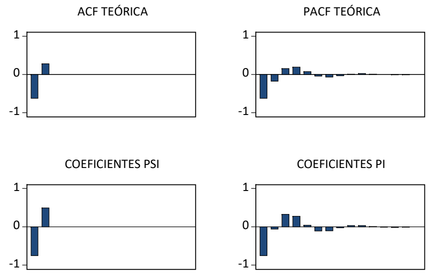
$\phi_1 = 0.75,\ \phi_2 = -0.5$

AR(2)



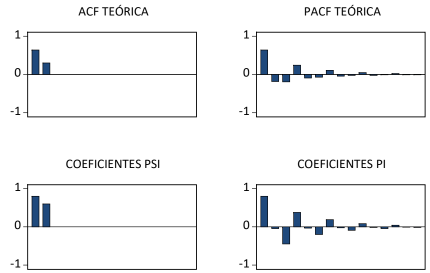
$\phi_1 = -0.8,\ \phi_2 = -0.6$

MA(2)



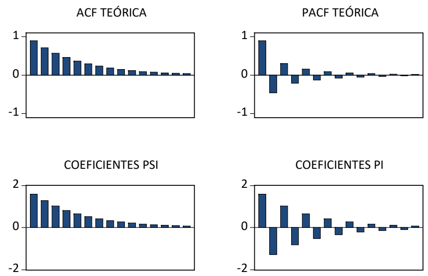
$\theta_1 = 0.75,\ \theta_2 = -0.5$

MA(2)



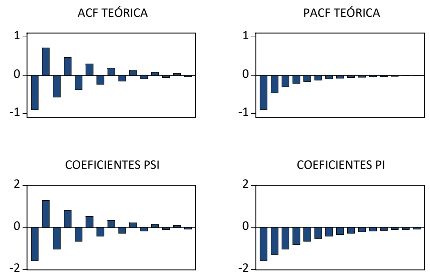
$\theta_1 = -0.8,\ \theta_2 = -0.6$

ARMA(1,1)



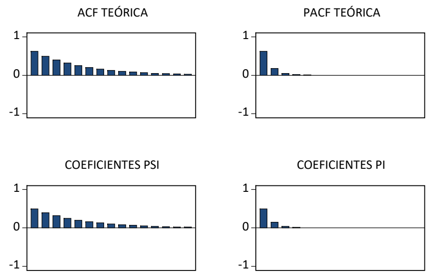
$\phi_1 = 0.8,\ \theta_1 = -0.8$

ARMA(1,1)



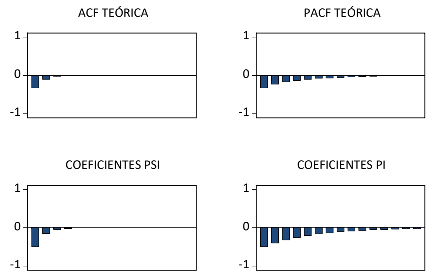
$\phi_1 = -0.8,\ \theta_1 = 0.8$

ARMA(1,1)



$\phi_1 = 0.8,\ \theta_1 = 0.3$

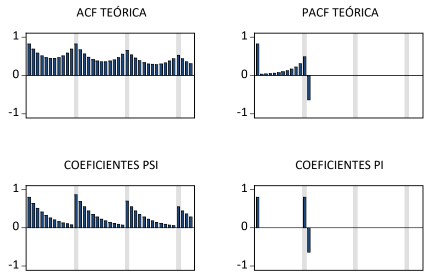
ARMA(1,1)



$\phi_1 = 0.3,\ \theta_1 = 0.8$

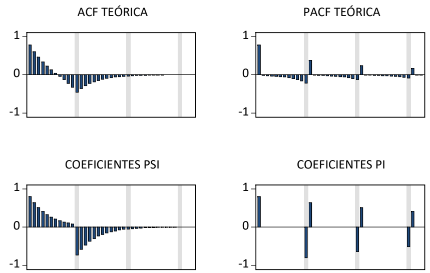
TABLA 2 - MODELOS ARMA MULTIPLICATIVOS ESTACIONARIOS E INVERTIBLES

$AR(1) \times AR(1)_{12}$



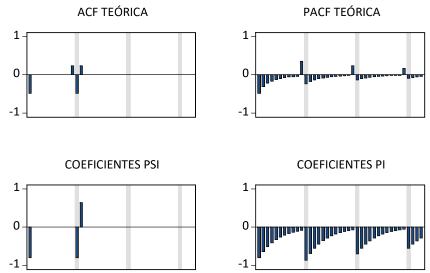
$\phi_1 = 0.8,\ \Phi_1 = 0.8$

$AR(1) \times MA(1)_{12}$



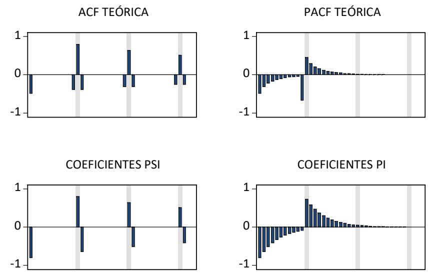
$\phi_1 = 0.8,\ \Theta_1 = 0.8$

$MA(1) \times MA(1)_{12}$



$\theta_1 = 0.8,\ \Theta_1 = 0.8$

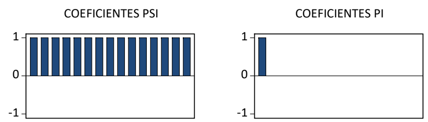
$MA(1) \times AR(1)_{12}$



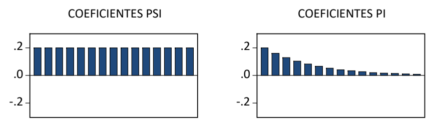
$\theta_1 = 0.8,\ \Phi_1 = 0.8$

TABLA 3 - MODELOS INVERTIBLES NO ESTACIONARIOS

PASEO ALEATORIO

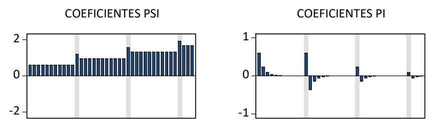


IMA(1,1)



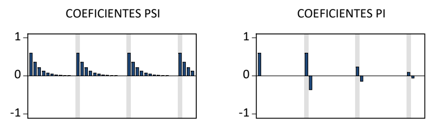
$\theta_1 = 0.8$

$IMA(1,1) \times IMA(1,1)_{12}$



$\theta_1 = 0.4,\ \Theta_1 = 0.4$

$AR(1) \times IMA(1,1)_{12}$



$\phi_1 = 0.6,\ \Theta_1 = 0.4$

$$\phi(B)\Phi(B^S)[\nabla^d\nabla_S^D Y_t' - \beta_0] = \theta(B)\Theta(B^S)A_t, \qquad [10]$$

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - ... - \phi_p B^p,$$
$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - ... - \theta_q B^q,$$
$$\Phi(B^S) = 1 - \Phi_1 B^S - \Phi_2 B^{2S} - ... - \Phi_p B^{PS},$$
$$\Theta(B^S) = 1 - \Theta_1 B^S - \Theta_2 B^{2S} - ... - \Theta_Q B^{QS},$$

todas las raíces de $\phi(x) = 0$, $\theta(x) = 0$, $\Phi(x) = 0$ y $\Theta(x) = 0$ fuera del círculo unitario,

$$\beta_0 = \mathrm{E}\left[\nabla^d\nabla_S^D Y_t'\right] \text{ (constante).}$$

REPRESENTACIÓN PSI GENERAL

[10] $\Rightarrow$
$$Y_t' = \frac{\beta_0}{\nabla^d\nabla_S^D} + \frac{\theta(B)\Theta(B^S)}{\phi(B)\Phi(B^S)\nabla^d\nabla_S^D} A_t, \text{ o bien} \qquad [11]$$

$$Y_t' = \beta_t + \psi(B)A_t, \text{ con} \qquad [11.1]$$

$$\beta_t \text{ tal que } \nabla^d\nabla_S^D\beta_t = \beta_0 \;\; (\Rightarrow \beta_t \text{ es una "tendencia determinista"}), \qquad [11.2]$$

$$\psi(B) = \frac{\theta(B)\Theta(B^S)}{\phi(B)\Phi(B^S)\nabla^d\nabla_S^D} = 1 + \psi_1 B + \psi_2 B^2 + ... = \sum_{i=0}^{\infty}\psi_i B^i \;\; (\psi_0 = 1). \qquad [11.3]$$

Aclaración 2: En [11.2], la expresión $\nabla^d\nabla_S^D\beta_t = \beta_0$ es una ecuación en diferencias cuya solución general es una función de t. Por ejemplo, con $d = 1$, $D = 0$, dicha expresión queda $\nabla\beta_t = \beta_0$, o $\beta_t - \beta_{t-1} = \beta_0$, cuya solución general es $\beta_t = a_0 + \beta_0 t$ (una "tendencia lineal determinista" con pendiente β_0).

[11.1]-[11.3] $\Rightarrow$
$$Y_t' = \beta_t + A_t + \psi_1 A_{t-1} + \psi_2 A_{t-2} + ..., \qquad [11.4]$$

$$\psi_i = \frac{\partial Y_t'}{\partial A_{t-i}} = \frac{\partial Y_{t+i}'}{\partial A_t} \;\; (i = 0, 1, 2, ...). \qquad [11.5]$$

REPRESENTACIÓN PI GENERAL

[10] $\Rightarrow$
$$\frac{\phi(B)\Phi(B^S)\nabla^d\nabla_S^D}{\theta(B)\Theta(B^S)} Y_t' = \frac{\phi(B)\Phi(B^S)}{\theta(B)\Theta(B^S)} \times \beta_0 + A_t, \text{ o bien} \qquad [12]$$

$$\pi(B)Y_t' = \mu_0 + A_t, \text{ con} \qquad [12.1]$$

$$\mu_0 = \frac{\phi(1)\Phi(1)}{\theta(1)\Theta(1)} \times \beta_0 \text{ (constante),} \qquad [12.2]$$

$$\pi(B) = \frac{\phi(B)\Phi(B^S)\nabla^d\nabla_S^D}{\theta(B)\Theta(B^S)} = 1 - \pi_1 B - \pi_2 B^2 - ... = -\sum_{i=0}^{\infty} \pi_i B^i \quad (\pi_0 = -1). \qquad [12.3]$$

[12.1]-[12.3] $\Rightarrow$ $\qquad Y_t' - \pi_1 Y_{t-1}' - \pi_2 Y_{t-2}' - ... = \mu_0 + A_t$, o bien

$$Y_t' = \mu_0 + \pi_1 Y_{t-1}' + \pi_2 Y_{t-2}' + ... + A_t, \qquad [12.4]$$

$$\pi_i = \frac{\partial Y_t'}{\partial Y_{t-i}'} = \frac{\partial Y_{t+i}'}{\partial Y_t'} \quad (i = 1, 2, ...). \qquad [12.5]$$

CÁLCULO DE LOS COEFICIENTES PSI - PI GENERALES

1. Calcular los coeficientes Φ_i^* $(i = 1, 2, ..., p^*)$ y Θ_i^* $(i = 1, 2, ..., q^*)$ de los polinomios

$$\Phi^*(B) = \phi(B)\Phi(B^S)\nabla^d\nabla_S^D = 1 - \textstyle\sum_{i=1}^{p^*} \Phi_i^* B^i, \text{ con } p^* = (p+d) + (P+D) \times S,$$

$$\Theta^*(B) = \theta(B)\Theta(B^S) = 1 - \textstyle\sum_{i=1}^{q^*} \Theta_i^* B^i, \text{ con } q^* = q + Q \times S.$$

2. Por un lado (ver [7] y [11.3]), calcular

$$\psi_i = -\Theta_i^* + \textstyle\sum_{j=1}^{p^*} \Phi_j^* \psi_{i-j}$$

para todo $i > 0$ (donde $\psi_0 = 1$, $\psi_i = 0$ si $i < 0$ y $\Theta_i^* = 0$ si $i > q^*$), y, por otro lado (ver [8] y [12.3]), calcular

$$\pi_i = \Phi_i^* + \textstyle\sum_{j=1}^{q^*} \Theta_j^* \pi_{i-j}$$

para todo $i > 0$ (donde $\pi_0 = -1$, $\pi_i = 0$ si $i < 0$ y $\Phi_i^* = 0$ si $i > p^*$).

OPERACIONES CON EVIEWS

Sección 19 pp. 85-94 de la guía *Introducción al Uso de EViews 4.1*.